Temperature dependent spin susceptibility in a two-dimensional metal.

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We consider a two-dimensional electron system with Coulomb interaction between particles at a finite temperature T. We show that the dynamic Kohn anomaly in the response function at $2k_F$ leads to a non-analytic, linear-in-T correction to the spin susceptibility, $\delta\chi(T) = AT$, same as in systems with short-range interaction. We show that the singularity of the Coulomb interaction at $q = 0$ does not invalidate the expansion of A in powers of r_s, but makes the expansion non-analytic. We argue that the linear temperature dependence is consistent with the general structure of Landau theory and can be viewed as originating from the non-analytic component of the Landau function near the Fermi surface.

Introduction — There has been substantial recent interest in the temperature dependence of various Fermi liquids properties for both short-range and long-range interactions between particles [1, 2, 3, 4, 5, 6]. The revival of interest in the problem is two-fold. On the experimental side, technical advances now allow one to measure the temperature dependence of the thermodynamic parameters such as specific heat and spin susceptibility in "classical" 2D Fermi liquids with short-range interaction, such as monolayers of ^{3}He, as well as study two-dimensional semiconductor structures with long-range interaction and with relatively low Fermi temperatures (~ 1 K). On the theory side, the leading interaction corrections turn out to be non-analytic functions of temperature making the subject particularly important.

Naïve power counting arguments suggest that the temperature dependence of any thermodynamic quantity, including the spin susceptibility and the specific heat coefficient $C(T)/T = \gamma$, should start with terms quadratic in temperature. This conjecture is based on the observation that a thermodynamic quantity at a finite temperature typically can be written as $\int a(\varepsilon)n(\varepsilon)d\varepsilon$, where $n(\varepsilon)$ is the Fermi distribution function and $a(\varepsilon)$ is some function. If the latter is smooth, the temperature dependence starts with a term of order T^2 [7]. Such a temperature correction is called "analytic." This is also consistent with the intuitive expectation of the one-to-one correspondence between the non-interacting Fermi gas and the interacting Fermi liquid since in the Fermi gas, the Sommerfeld expansion leads to simple quadratic temperature corrections.

However, the assumption about the analyticity of the functions involved in the calculation of various thermodynamic properties of the Fermi liquid is quite generally not justified because in any Fermi liquid, the dynamic interaction between particles gives rise to a non-analytic energy dependence of $a(\varepsilon)$. This leads to temperature corrections which do not scale as T^2 and are therefore called "non-analytic." Collecting these non-analytic corrections is a subtle theoretical problem.

The subject of this paper is the temperature correction to the spin susceptibility for a 2D system of fermions interacting via a long-range Coulomb interaction. For Fermi systems with short-range interaction, perturbative calculations for a model with a small $U(q)$ have demonstrated that $\delta\chi_s(T)$ is linear in T and that the prefactor depends only on $U(2p_F)$. This result, however, was obtained under the assumption that for all q, the dimensionless Born parameter $u(q) = mU(q)/(2\pi)$ is small. For Coulomb interaction this is obviously not the case, and one has to verify explicitly whether the linear dependence $\delta\chi_s(T) \propto T$ still holds, and whether the prefactor can be expanded in r_s. The spin susceptibility has recently been measured at various T in Si inversion layers [8, 9, 10, 11], and a quantitative theory is required to interpret the temperature dependence of the experimentally measured χ_s. In what follows, we compute the spin susceptibility in the perturbation theory and beyond, and relate the prefactor of the linear in T term in an arbitrary Fermi liquid to the spin component of the quasiparticle scattering amplitude at the scattering angle $\theta = \pi$.

We also consider in detail the relation between the non-analytic T dependence of the thermodynamic parameters and Landau Fermi liquid theory. The Landau theory operates with the quasiparticle interaction function for the particles at the Fermi surface. In this theory, the spin susceptibility is independent of T, and is expressed via the particular partial component of the Landau function. The temperature corrections to the Fermi liquid theory come from quasiparticles which are slightly off the Fermi surface. We demonstrate explicitly that non-analytic corrections to the spin susceptibility can be viewed as originating from the non-analytic momentum dependence of the quasiparticle interaction function $f(p, p')$ at small deviations from the Fermi surface. We argue that $f(p, p')$ is non-analytic in deviations from the Fermi surface, and that this non-analyticity gives rise to the emergence of the linear in T terms in the $g-$factor and the spin susceptibility.

Perturbation theory — Consider first the case when the Coulomb interaction is weak at $q \sim p_F$, i.e., when $r_s \ll 1$. The temperature correction to the spin susceptibility can be calculated either by explicitly evaluating the static particle-hole polarization bubble in a zero field, with insertions due to the interaction, or by evaluating the free energy in a finite magnetic field H and then differentiating over H. Either way, one obtains that, to the leading (second) order in the interaction, the linear in T term in the spin susceptibility comes from

$2p_F$ processes, and

$$\delta\chi_s(T) = \chi_{\text{Pauli}} \left(\frac{r_s}{4}\right)^2 \frac{T}{E_F}, \quad \text{for } T/E_F \ll 1, \tag{1}$$

where $\chi_{\text{Pauli}} = \mu_B^2 m/\pi$ is the susceptibility of free fermions, and $r_s = \sqrt{2}mU(2p_F)/\pi$. The special role of $2p_F$ terms in the perturbation theory can be most easily understood by evaluating $\delta\chi_s(T)$ from the free energy. To second order in the interaction, the free energy Ω_2 consists of two particle-hole bubbles connected by $U(q)$

$$\Omega_2 \propto U^2(2p_F)T \sum_{\Omega_m} \sum_{\alpha\beta;\gamma\delta} \int d^2q \Pi^{\alpha,\beta}(q,\Omega_m)\Pi^{\gamma\delta}(q,\Omega_m), \tag{2}$$

where α, β, γ and δ are spin components of four fermions involved. Since the Coulomb interaction is spin independent, the spins of the two fermions within each bubble are parallel (i.e., $\alpha = \beta$ and $\gamma = \delta$). However the spins in different bubbles can be either parallel or antiparallel to each other. The singular $\delta\chi_s(T)$ comes from the antiparallel spin configuration between bubbles. For such spin orientation, fermions in each bubble have different Fermi momenta p_F^+ and p_F^- due to the Zeeman splitting. Near $2p_F$, this splitting is relevant as the polarization bubble is non-analytic in both momentum and frequency. The non-analytic momentum dependence is normally associated with the Kohn anomaly and related Friedel oscillations. For the T dependence of the spin susceptibility, however, one actually needs the dynamic polarization bubble. In 2D, the singular part of the polarization bubble behaves near $2p_F$ as [12]

$$\Pi(q,\Omega_m) \propto \sqrt{\left(\frac{q}{2p_F} + i\frac{\Omega_m}{v_F q}\right)^2 - 1} + \sqrt{\left(\frac{q}{2p_F} - i\frac{\Omega_m}{v_F q}\right)^2 - 1}. \tag{3}$$

At small frequencies and $q < 2p_F$, this reduces to

$$\Pi(q,\Omega_m) \propto \frac{|\Omega_m|}{\sqrt{2p_F - q}}. \tag{4}$$

Integrating the product $\Pi^{++}(q,\Omega_m)\Pi^{--}(q,\Omega_m)$ in (2) over q, we find that the frequency dependence is not analytic: $\Omega_2 \propto U^2(2p_F)T\sum_{\Omega_m}\Omega_m^2 \log[\Omega_m^2 + (\mu_B H)^2]$. Evaluating the sum one finds that Ω_2 contains a cross term TH^2 [13]. Differentiating over frequency, one then obtains $\delta\chi_s(T) \propto T$, as in (1).

At the same time, the potentially dangerous small q region, where the Coulomb interaction is large, does not contribute to (1) (in this respect, our results differ from those in Ref [6]). The reason is that at small q each of the two polarization bubbles contains only a non-singular, multiplicative dependence on the magnetic field; in two dimensions, this dependence comes through $v_F(H) = p_F(H)/m$ in $\Pi(q,\Omega_m) = (m/2\pi)\left[1 - |\Omega_m|/\sqrt{\Omega_M^2 + (v_F q)^2}\right]$. This multiplicative dependence implies that the magnetic field only accounts for regular corrections in the form $(\mu_B H/E_F)^2$ for the $q = 0$ piece in the free energy, i.e., no crossed TH^2 term appears.

Higher-order terms — As we just found, the second-order result for $\delta\chi_s(T)$ does not distinguish between short-range and long-range interaction, i.e., the specifics of the Coulomb case does not show up. There is no guarantee, however, that this will remain so beyond the second order. Of particular interest is whether the divergence of the Coulomb interaction at $q = 0$ affects the expansion of $\delta\chi_s(T) \propto T$ in powers of r_s. For this, we computed the corrections to (1) from the third-order diagrams. We found that the divergent $U(0)$ still does not directly contribute to the spin susceptibility, however the prefactor of the linear in T term gets modified due to vertex corrections to $U(2p_F)$. The corrections involve $U(2p_F)$ itself and the momentum integrals of the interaction potential. The most singular of these corrections accounts for the renormalization between $U(2p_F)$ and the spin component of the Landau function $\Gamma_s(\pi)$. This renormalization involves $T\sum_\omega \int d^2p U(k-p)G(p,\omega)G(p-2k,\omega)$ and yields the multiplicative correction to (1) in the form

$$B = 1 + \frac{4}{\pi}\int_0^\pi d\theta\, u_\theta\, \cos\frac{\theta}{2}\, \log\frac{\sqrt{1+\sin\theta/2} + \sqrt{1-\sin\theta/2}}{\sqrt{1+\sin\theta/2} - \sqrt{1-\sin\theta/2}}, \tag{5}$$

where $u(\theta) = (m/2\pi)U(q = 2p_F\sin\theta/2)$. One can easily verify that for short-range $u(\theta)$, the integral over θ converges, but for the Coulomb interaction, when $u(\theta) \propto 1/\sin(\theta/2)$, the integral is confined to small θ and diverges as $\log^2$. The true divergence is indeed cut off by the screening effects. Still, it implies that in contrast to a short-range potential, the prefactor for the linear in T term in the spin susceptibility in the 2D Coulomb system is non-analytic in r_s. Including screening in the usual way, we obtain from (5) within logarithmic accuracy

$$B = 1 + \frac{\sqrt{2}r_s}{\pi}\left[\log^2 r_s + O(\log r_s) + ...\right], \tag{6}$$

In a generic Fermi liquid, the full linear in T correction to the spin susceptibility is evaluated in the same way as the correction to the specific heat [5]. At each order of perturbation, one selects two bubbles in which one keeps the singular frequency dependence of $\Pi(q,\Omega_m)$, and evaluate all other bubbles at Ω_0. The two selected bubbles yield the TH^2 term, other bubbles contribute to the prefactor via the renormalization of the $2k_F$ vertex. Extending the analysis in [5] to the spin susceptibility we obtain that the prefactor is expressed in terms of the spin component of the full quasiparticle scattering amplitude at the scattering angle $\theta = \pi$. In the explicit form,

$$\delta\chi_s(T) = \chi_{\text{Pauli}}\,\frac{T}{2E_F}\left[\frac{m^*}{m}F_{\text{sp}}(\pi)\right]^2. \tag{7}$$

Here $E_F = v_F p_F/2$ is the Fermi energy for free fermions, m^* is the effective mass, and $F_{\text{sp}}(\pi)$ is the spin component of the scattering amplitude. At weak coupling, $F_{\text{sp}}(\pi) = -mU(2p_F)/(2\pi) = -r_s/(2\sqrt{2})$, and Eq. (7) reduces to Eq. (1). In a generic Fermi liquid, $F_{\text{sp}}(\pi) = \sum_{n=0}(-1)^n(2n +$

1) $f_{\text{sp},n}/(1+f_{\text{sp},n})$, where $f_{\text{sp},n}$ are the partial spin components of the Landau function.

If the system is close to a ferromagnetic (Stoner) instability, $f_{\text{sp},0} \approx -1$, and $F^2_{\text{sp}}(\pi)$ can be well approximated by $f^2_{\text{sp},0}/(1+f_{\text{sp},0})^2$ Then $\delta\chi_s(T) \approx (T/2E_F)\chi_s^2/\chi_{\text{Pauli}}$, where $\chi_s = \chi_{\text{Pauli}}(m^*/m)/(1+f_{\text{sp},0})$ is the spin susceptibility in a Fermi liquid at $T=0$. We did not analyze higher-order terms in temperature, but based on the form of $\delta\chi_s(T)$, it is tempting to assume that

$$\chi_s^{-1}(T) = \chi_s^{-1}(T=0) - \frac{T}{2E_F}\chi^{-1}_{\text{Pauli}}. \tag{8}$$

If, on the contrary, the spin component of the scattering amplitude is small, but m^*/m is arbitrary, as some studies suggest [9], the same consideration yields

$$\chi_s^{-1}(T) = \chi_s^{-1}(T=0) - \frac{T}{2E_F}F_s^2(\pi)\chi^{-1}_{\text{Pauli}}. \tag{9}$$

In silicon inversion layer, $E_F \sim 6K$ for typical densities [9]. The susceptibility measurements have been reported for $T \sim 2-4K$. $\chi_s^{-1}(T)$ measured in units of χ_{Pauli} changes by about 20% between $2K$ and $4K$ [9]. This would be consistent with Eq. (8), however the sign of the measured temperature correction is opposite to that in (8). Recent Shubnikov-deHaas measurements, however, reported a much weaker, almost undetectable T dependence of $\chi_s(T)$, from which $\delta\chi_s(T)$ could not be extracted [14]. This much weaker effect would be more consistent with Eq. (9) if we assume that $F_{\text{sp}}(\pi)$ remains small. More precise measurements of $\chi_s(T)$ are clearly called for to test our theoretical predictions.

Extended Landau formalism — We now consider in more detail the physics behind the linear in T dependence of the spin susceptibility. We argue that this term is actually consistent with the structure of Landau theory can be viewed as originating from the non-analytic structure of the Landau function near the Fermi surface.

We remind that the Landau function is the second variational derivative of the energy of the system with respect to the distribution function of quasiparticles $n_\sigma(\mathbf{p})$

$$f_{\sigma\sigma'}(\mathbf{p},\mathbf{p}') = \frac{\delta^2 E}{\delta n_\sigma(\mathbf{p})\delta n_\sigma(\mathbf{p}')}. \tag{10}$$

The energy gain of a quasiparticle in a weak external magnetic field $\mathbf{H}$ is [7]

$$\delta\varepsilon(\mathbf{p}) = -\mu_{\text{B}}(\boldsymbol{\sigma}\mathbf{H}) + \text{Tr}_{\sigma'}\int f_{\sigma\sigma'}(\mathbf{p},\mathbf{p}')\frac{\partial n(\mathbf{p}')}{\partial\varepsilon'}\delta\varepsilon(\mathbf{p}')\frac{d^2\mathbf{p}'}{(2\pi)^2}, \tag{11}$$

where both $\delta\varepsilon$ and δn are matrices in the spin space. The g-factor of a quasiparticle with momentum $\mathbf{p}$ is defined by

$$\delta\varepsilon(\mathbf{p}) = -\frac{g(\mathbf{p})\mu_{\text{B}}}{2}(\boldsymbol{\sigma}\boldsymbol{H}), \tag{12}$$

and the spin susceptibility of a Fermi liquid is expressed as the momentum integral over $g(\mathbf{p})$

$$\chi = -\frac{\mu_{\text{B}}^2}{4}\int\frac{\partial n(\mathbf{p})}{\partial\varepsilon}g(\mathbf{p})\frac{d^d\mathbf{p}}{(2\pi)^d}, \tag{13}$$

Eqs. (11) and (12) determine the integral equation for the g-factor [7]:

$$g(\mathbf{p}) = 2 + \frac{1}{2}\int f_{\text{sp}}(\mathbf{p},\mathbf{p}')\frac{\partial n(\mathbf{p}')}{\partial\varepsilon'}g(\mathbf{p}')\frac{d^d\mathbf{p}'}{(2\pi)^d}. \tag{14}$$

Here f_{sp} is the spin component of the Landau function: $\nu\hat{f}(\mathbf{p},\mathbf{p}') = f_c\hat{I} + \boldsymbol{\sigma}\boldsymbol{\sigma}' f_{\text{sp}}(\mathbf{p},\mathbf{p}')$, where $\nu = m^*/\pi$.

At zero temperature, the integration in (13) is confined to the Fermi surface. At finite T, the quasiparticles are allowed to deviate from the Fermi surface. Introducing $\xi = v_{\text{F}}(p-p_{\text{F}})$ and $\xi' = v_{\text{F}}(p'-p_{\text{F}})$, and assuming that $g(\mathbf{p})$ depends on ξ but not on the direction of $\mathbf{p}$, we re-write Eqs. (14) and (13) as

$$g(\xi) = 2 + \int\overline{f_{\text{sp}}}(\xi,\xi')\frac{\partial n}{\partial\xi'}g(\xi')d\xi' \tag{15}$$

and

$$\chi = -\frac{\mu_{\text{B}}^2\nu}{2}\int\frac{\partial n}{\partial\xi}g(\xi)d\xi. \tag{16}$$

Here $\overline{f_{\text{sp}}}(\xi,\xi')$ is the interaction function averaged over the angle ϕ between the momenta $\mathbf{p}$ and $\mathbf{p}'$: $\overline{f_{\text{sp}}}(\xi,\xi') = \int_0^{2\pi} f_{\text{sp}}(\xi,\xi';\phi)\,d\phi/(2\pi)$.

At $T=0$, only particles at the Fermi surface matter, and Eq. (15) yields the well-known result $g(T=0) = 2/(1+f_{\text{sp},0})$ For calculations at a finite T, we need the solution at small but finite ξ. To illustrate the appearance of singular terms, let us first study the structure of the finite temperature f-function in the limit of weak interactions. The corresponding RPA correlation energy can be written as follows [15]

$$E_{\text{cor}} = \text{Re}\int\frac{de^2}{e^2}\int\frac{d\omega}{2\pi i}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\Pi(\omega,\mathbf{q})\left[V(\omega,\mathbf{q}) - v(\mathbf{q})\right]. \tag{17}$$

In Eq. (17), $v(q)$ is the bare Coulomb interaction in two dimensions and $V(\omega,\mathbf{q})$ is the dynamically screened interaction $V(\omega,\mathbf{q}) = v(\mathbf{q})\left[1-\Pi(\omega,\mathbf{q})v(\mathbf{q})\right],^{-1}$. The polarizability $\Pi(\omega,\mathbf{q})$ is defined as

$$\Pi(\omega,\mathbf{q}) = \text{Tr}_\sigma\int\frac{d\varepsilon}{2\pi i}\int\frac{d^2\mathbf{p}}{(2\pi)^2}G_\sigma(\varepsilon,\mathbf{p})G_\sigma(\varepsilon+\omega,\mathbf{p}+\mathbf{q}) \tag{18}$$

where $G_\sigma(\varepsilon,\mathbf{p})$ is the time-ordered Green's function, which we write as a functional of the quasiparticle distribution function:

$$G_\sigma(\varepsilon,\mathbf{p}) = \frac{n_\sigma(\mathbf{p})}{\varepsilon - E_\sigma(\mathbf{p}) - i0} + \frac{1-n_\sigma(\mathbf{p})}{\varepsilon - E_\sigma(\mathbf{p}) + i0}. \tag{19}$$

Using the definition (10) and Eqs. (17), (18), and (19), one can obtain the quasiparticle interaction function by a straightforward evaluation of the derivatives with respect to $n_\sigma(p)$ [16]. At finite temperatures, we have to allow quasiparticles to depart from the Fermi surface. Keeping this in mind, we find that the spin-dependent part of the Landau function is essentially the dynamically screened Coulomb interaction, determined by the values of momentum and energy transfer of the interacting quasiparticles:

$$f_{\text{sp}}^{(\text{RPA})}(\mathbf{p},\mathbf{p}') = -\text{Re}\,V[E(\mathbf{p}) - E(\mathbf{p}'),\mathbf{p}-\mathbf{p}']. \tag{20}$$

In the limit of low temperatures $T/E_{\rm F} \ll r_{\rm s} \ll 1$, we can present the spin-dependent part of the Landau function $f(\mathbf{p},\mathbf{p}') = f_{\rm sp}(\xi,\xi';\phi)$ as $f_{\rm sp}(\xi,\xi';\phi) = f_{\rm reg}(\xi,\xi';\phi) + f_{\rm sing}(\xi,\xi';\phi)$, where the "regular" part is due to the statically screened Coulomb interaction, and the "singular" part comes the singular *dynamic* screening near $2k_F$. A small ξ,ξ', the singular part is a small correction to $f_{\rm reg}$ and can be written as

$$f_{\rm sing}(\xi,\xi';\phi) = -\frac{1}{\nu}\left[f_{\rm reg}(\xi,\xi';\phi)\right]^2 \Pi_{\rm sing}(\xi,\xi';\phi), \tag{21}$$

where $\Pi_{\rm sing}$ is the dynamic part of Π, given by Eq. (3) at $\omega = E(p) - E(p')$.

For actual calculations of the susceptibility, we will need the interaction function averaged over the angle ϕ. Performing the calculations, we find that the singular part is non-analytic in the deviations from the Fermi surface.

$$\overline{f_{\rm sing}}(\xi,\xi') = f_{\rm sp,0} - \frac{(f_{\rm sp,0})^2}{2}\left(\frac{m^*}{m}\right)\frac{|\xi-\xi'|}{E_F} + \dots, \tag{22}$$

where the dots stay for regular terms in ξ^2 and $(\xi')^2$. We explicitly verified that the non-analyticity in (22) originates from the dynamic $2k_F$ Kohn anomaly. This agrees with our diagrammatic analysis above.

Substituting the non-analytic part of the Landau function into the Eq. (15) for the g-factor, we obtain the following integral equation:

$$\begin{aligned} g(\xi) = {}& 2 + f_{\rm sp,0}\int_{-E_{\rm F}}^{+\infty} g(\xi')\frac{\partial n}{\partial \xi'}d\xi' \\ & - \frac{f_{\rm sp,0}^2}{8E_{F,0}}\left(\frac{m^*}{m}\right)\int_{-E_{\rm F}}^{+\infty}\left|\xi-\xi'\right| g(\xi')\frac{\partial n}{\partial \xi'}d\xi', \end{aligned} \tag{23}$$

This integral equation can be solved by iterations, using the zero temperature result $g(T=0) = g^* = 2/(1+f_{\rm sp,0})$ as the first approximation. Performing the calculations, we obtain

$$\begin{aligned} g(\xi,T) = {}& g^*\left[1 + f_{\rm sp,0}^2\frac{m^*}{m}\ (g^*-2)\,\frac{T}{8E_{\rm F}}\right] \\ & + f_{\rm sp,0}^2\frac{m^*}{m}\ \frac{T}{8E_F}\left[\frac{\xi}{T} + 2\ln\left(1+e^{-\xi/T}\right)\right], \end{aligned} \tag{24}$$

Using Eqs. (16) we then obtain for the spin susceptibility

$$\chi = \frac{\mu_{\rm B}^2\nu g^*}{2}\left[1 - g^* f_{\rm sp,0}^2\frac{m^*}{m}\ \frac{T}{8E_{\rm F}}\right]. \tag{25}$$

This agrees with the result of the diagrammatic treatment, Eq. (7), for the case when the full scattering amplitude $F_{\rm sp}(\pi)$ can be approximated by its zeroth partial component $F_{\rm sp}^2 \approx f_{\rm sp,0}^2/(1+f_{\rm sp,0})^2$. This approximation is implicit in the RPA formalism. In a more generic analysis, one indeed should recover the full scattering amplitude.

We see therefore that the linear in T correction to the spin susceptibility can be understood as originating from the non-analytic momentum dependence of the Landau function at small deviations from the Fermi surface. This non-analytic momentum dependence $f_{\rm sing} \propto |\xi-\xi'|$ is the fundamental consequence of the dynamic $2p_F$ Kohn anomaly in a generic Fermi liquid.

To conclude, we considered the temperature dependence of the spin susceptibility in a 2D electron system with Coulomb interaction. We found that the leading temperature correction $\delta\chi_s(T)$ is linear in T and comes from the $2p_F$ singularity in the dynamical response function (the dynamic Kohn anomaly). The origin of the effect is the same as in systems with short-range interaction. However, for Coulomb interaction, the prefactor of the $O(T)$ term is itself non-analytic in r_s. We also analyzed the emergence of the $O(T)$ term in $\delta\chi_s(T)$ by extending the Landau formalism to finite T. We demonstrated that within this approach, the non-analytic temperature dependence of the susceptibility originates from the non-analytic momentum dependence of the Landau function at small deviations from the Fermi surface. The experimental verification of the linear in T dependence of the spin susceptibility is clearly called for.

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